Heterogeneity and Robustness in Social Learning

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Heterogeneity in Biological & Human Systems

Threshold Response Models

Different Thresholds

Division of Labour

Different Specialisms

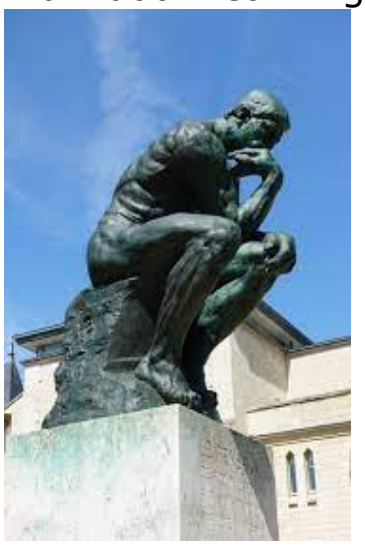
Behaviour Types

Shy vs Bold

Social vs Individual Learning

Individual Learning

Lone Reflection & Discovery



Rodin: Thinker

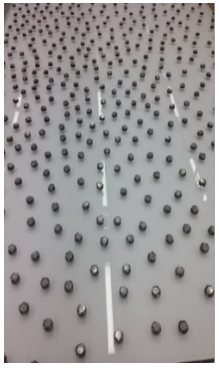
Social Learning

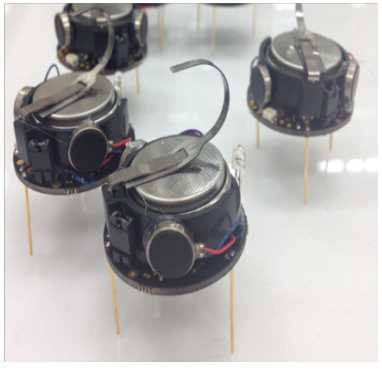
Discussion, Disagreement & Consensus

Raphael: The School of Athens

Waggle Dance

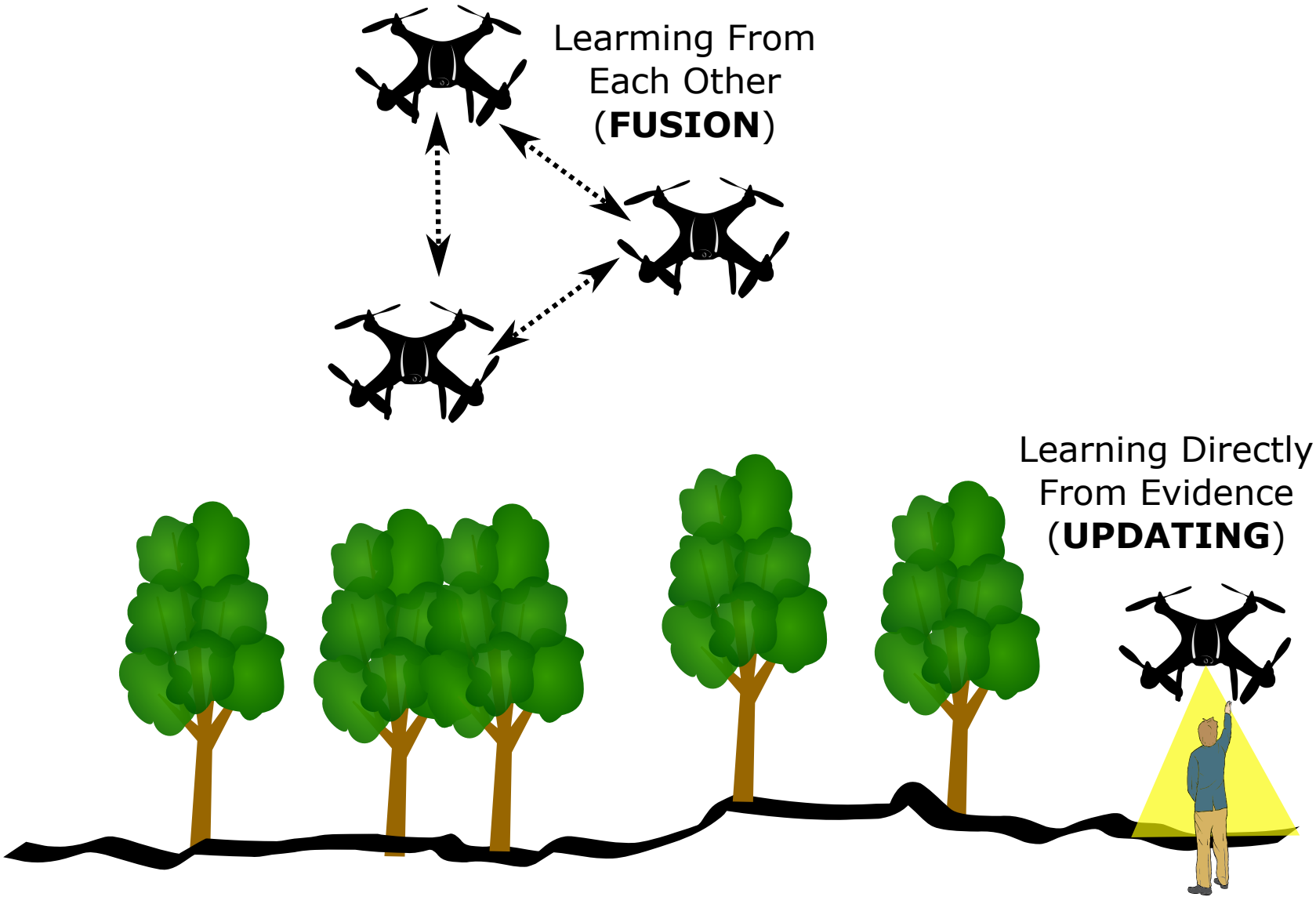
Kilobots





Best-of-N Problem

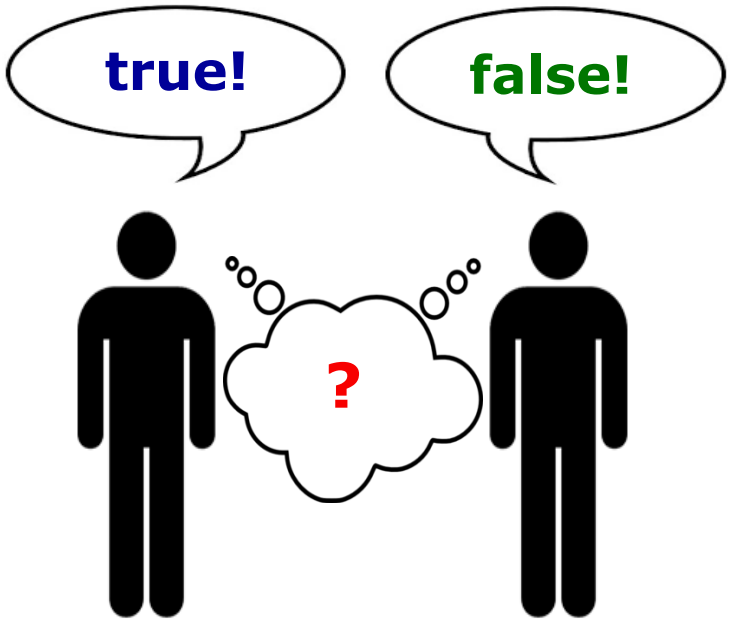
The Two Processes View of Social Learning



True, False & Uncertain: A Route to Consensus

- Consider a simple social learning problem where a population must determine whether a hypothesis H is true or false.
- At any time each individual is in one of three belief states: true (**t**), uncertain (**u**) or false (**f**).

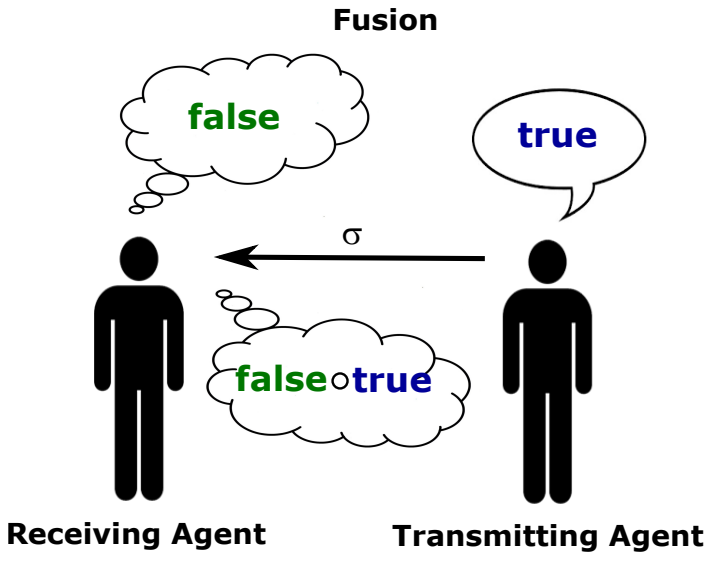
Disagreement Leads to Doubt

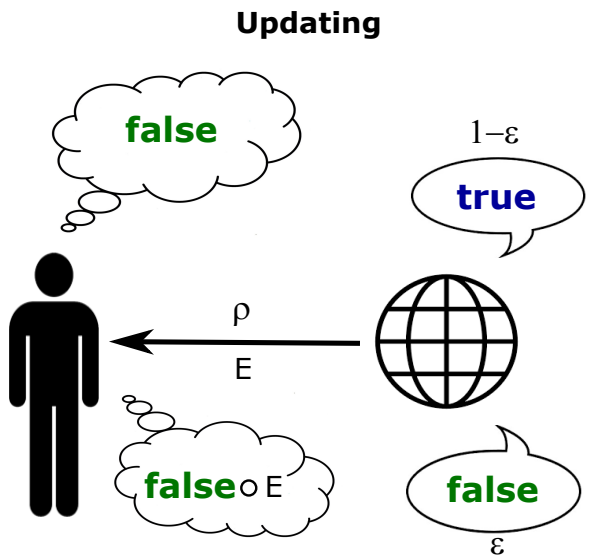


Undecided Individuals Help Consensus in Fish Schools

A Simple Two Process Model

- The aim is to learn whether a hypothesis is true or false.
- We assume that the hypothesis is in fact true
- Key parameters: σ (fusion rate), ρ (evidence rate), ϵ (noise).

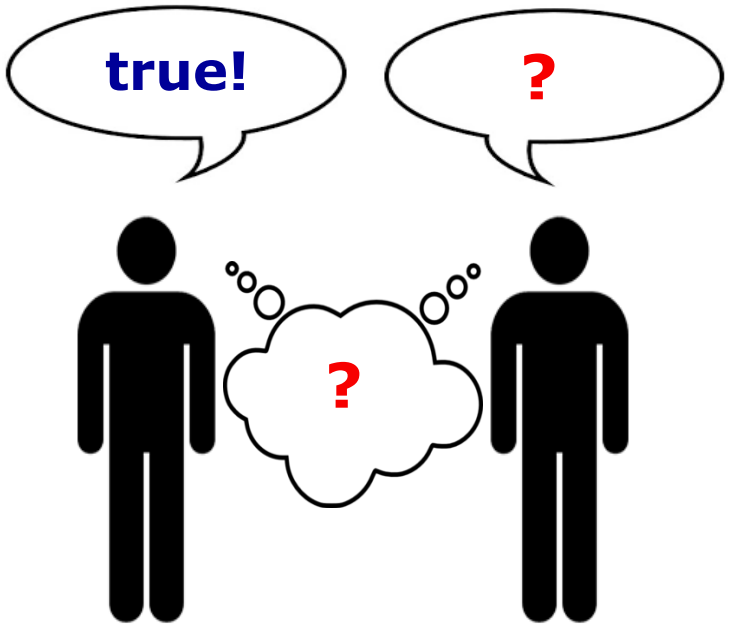




- $\mathbf{P}_t = (P_t(\mathbf{f}), P_t(\mathbf{u}), P_t(\mathbf{t}))$ denotes the proportion of agents in each belief state at time t
- Difference Equation: $\mathbf{P}_{t+1}^T = U(F^{\mathbf{P}_t}\mathbf{P}_t^T)$

Behaviour Types: Cautious Fusion

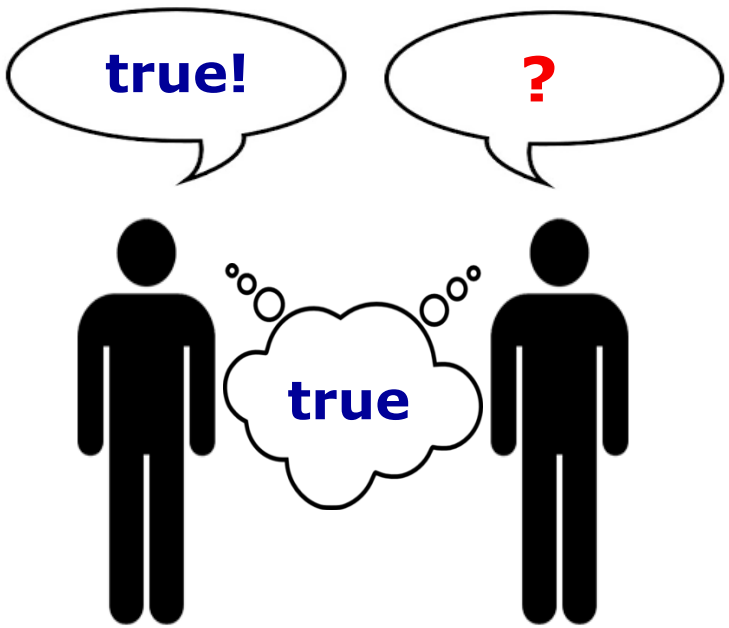
Uncertainty dominates over certainty so that if a receiving agent with committed belief state **t** or **f** interacts with a transmitting agent with uncertain truth state **u** they will abandon their committed position and change their belief to **u**



Agent 1 \ Agent 2	f	u	t
f	f	u	u
u	u	u	u
t	u	u	t

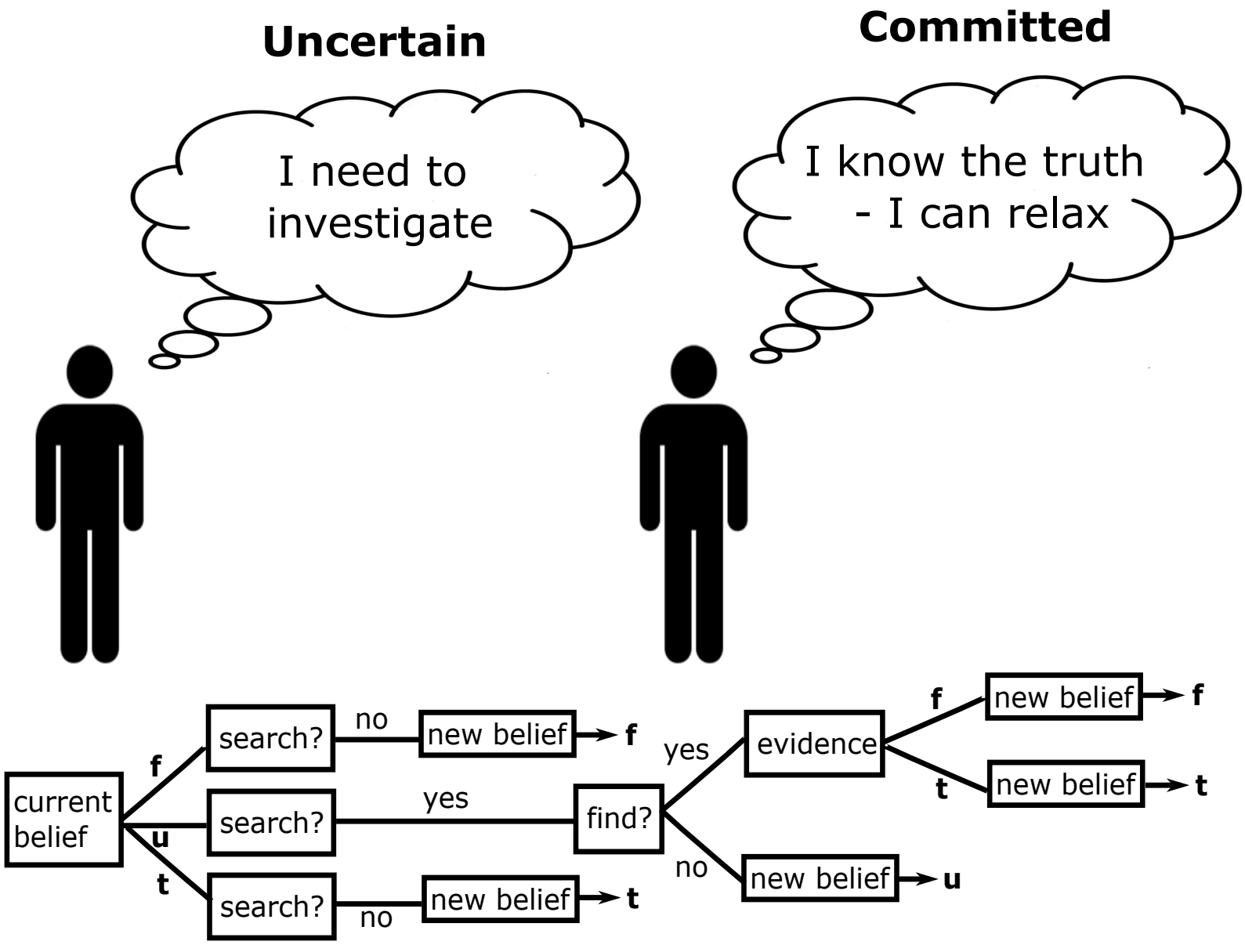
Behaviour Types: Adventurous Fusion

Certainty dominates over uncertainty so that if the receiving agent with belief state **u** interacts with a transmitting agent with committed belief states **t** or **f**, they adopt the latter.

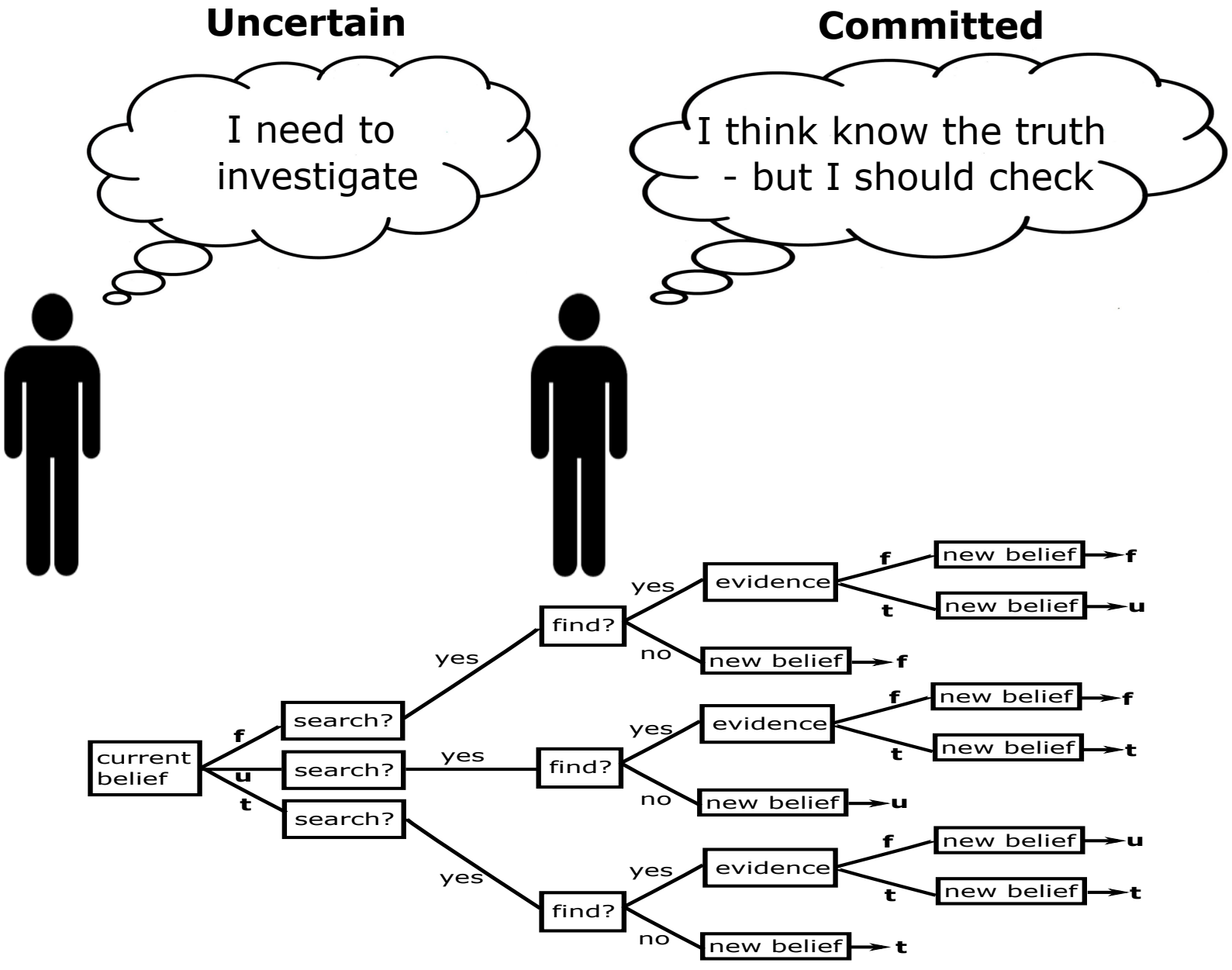


Agent 1 \ Agent 2	f	u	t
f	f	f	u
u	f	u	t
t	u	t	t

Behaviour Types: Confident Updating



Behaviour Types: Inquisitive Updating



Four Behaviour Types for Social Learning

Cautious: $F_C^{\mathbf{P}} = \begin{pmatrix} 1-\sigma+\sigma P(\mathbf{f}) & 0 & 0 \\ \sigma(1-P(\mathbf{f})) & 1 & \sigma(1-P(\mathbf{t})) \\ 0 & 0 & 1-\sigma+\sigma P(\mathbf{t}) \end{pmatrix}$

Adventurous: $F_A^{\mathbf{P}} = \begin{pmatrix} 1-\sigma P(\mathbf{t}) & \sigma P(\mathbf{f}) & 0 \\ \sigma P(\mathbf{t}) & 1-\sigma(P(\mathbf{t})+P(\mathbf{f})) & \sigma P(\mathbf{f}) \\ 0 & \sigma P(\mathbf{t}) & 1-\sigma P(\mathbf{f}) \end{pmatrix}$

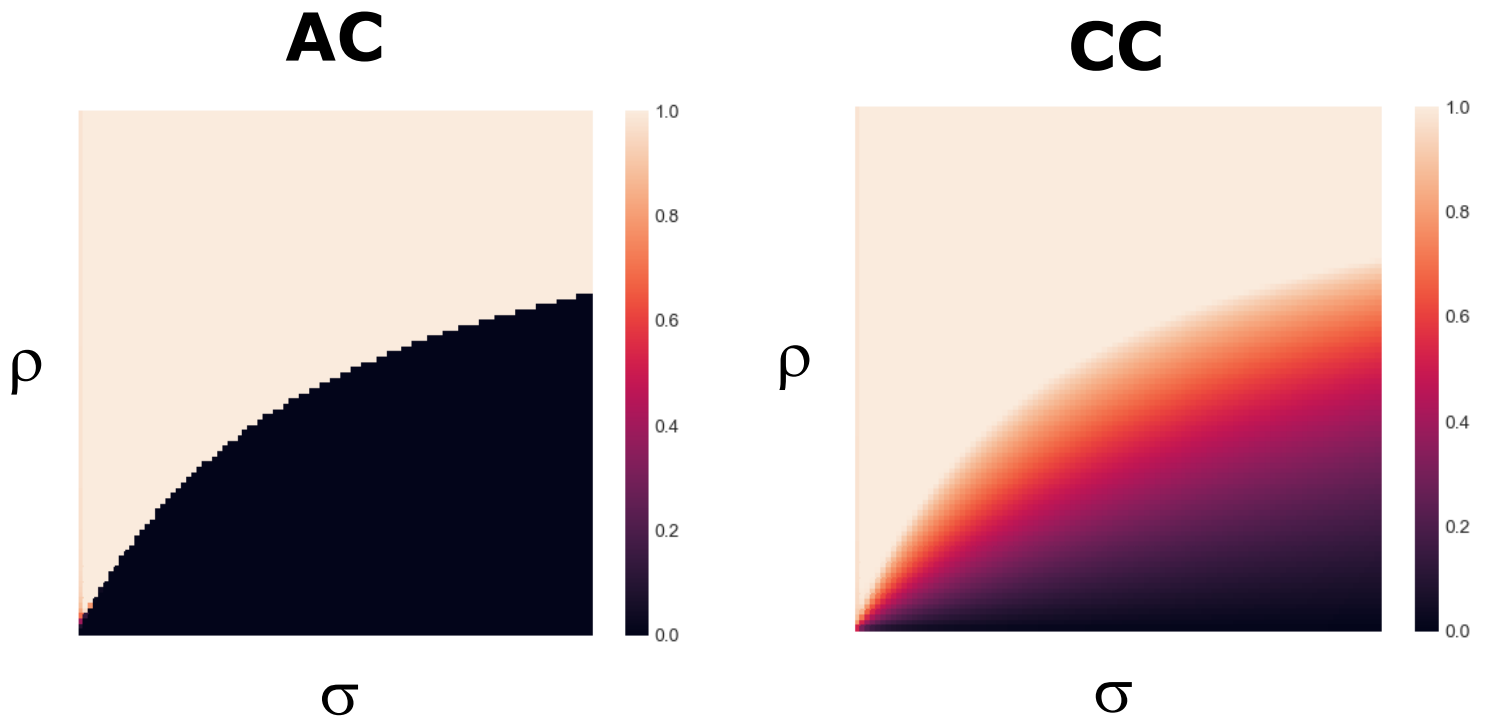
Confident: $U_C = \begin{pmatrix} 1 & \rho\epsilon & 0 \\ 0 & 1-\rho & 0 \\ 0 & \rho(1-\epsilon) & 1 \end{pmatrix}$

Inquisitive: $U_I = \begin{pmatrix} 1-\rho+\rho\epsilon & \rho\epsilon & 0 \\ \rho(1-\epsilon) & 1-\rho & \rho\epsilon \\ 0 & \rho(1-\epsilon) & 1-\rho\epsilon \end{pmatrix}$

	Cautious	Adventurous
Confident	**CC** :$\mathbf{P}_{t+1}^T = U_C(F_C^{\mathbf{P}_t}\mathbf{P}_t^T)$	**AC**: $\mathbf{P}_{t+1}^T = U_C(F_A^{\mathbf{P}_t}\mathbf{P}_t^T)$
Inquisitive	**CI**: $\mathbf{P}_{t+1}^T = U_I(F_C^{\mathbf{P}_t}\mathbf{P}_t^T)$	**AI**: $\mathbf{P}_{t+1}^T = U_I(F_A^{\mathbf{P}_t}\mathbf{P}_t^T)$

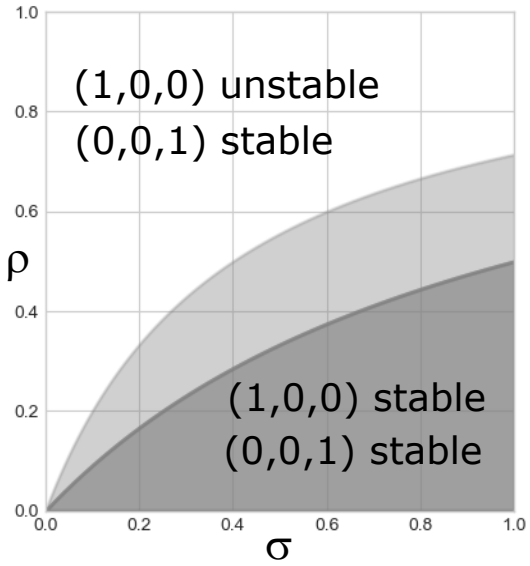
Adventurous vs Cautious Fusion

- Assume updating is confident: **AC** vs **CC**
- Initial Proportion: $\mathbf{P}_0 = (0.9, 0, 0.1)$

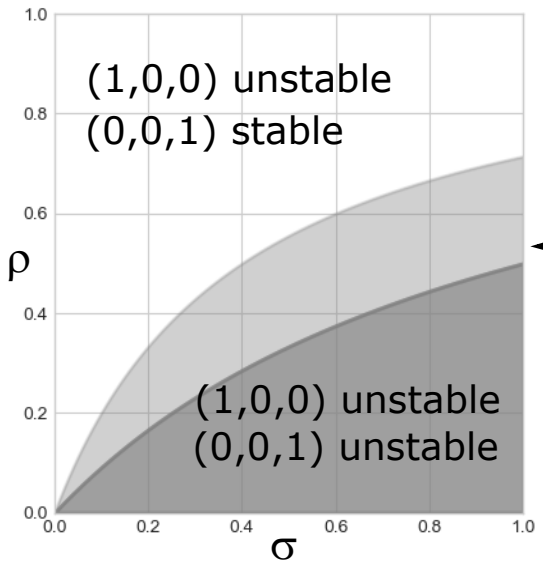


Fixed Points & Mixtures

AC

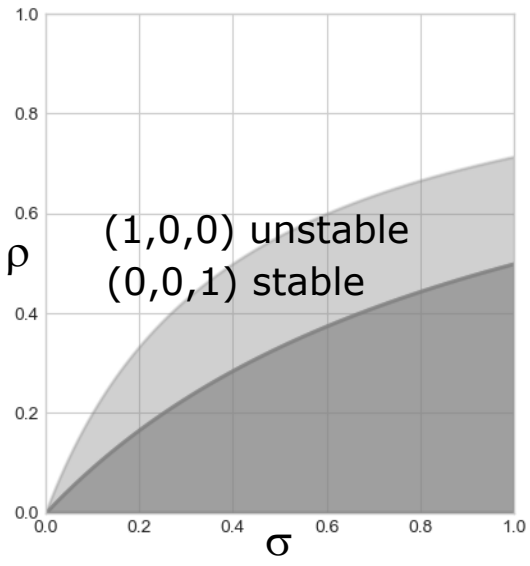


CC

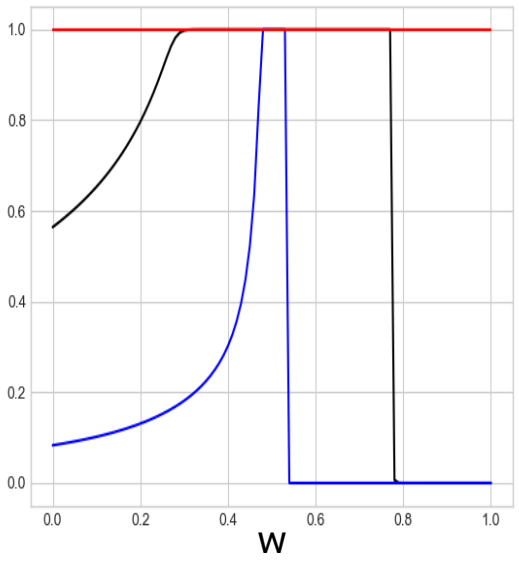


$$\rho = \frac{\sigma}{1+\sigma-2\epsilon}$$

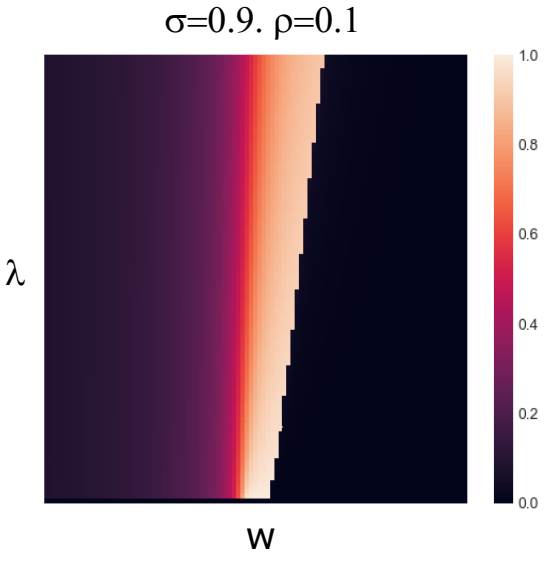
0.5 AC & 0.5 CC

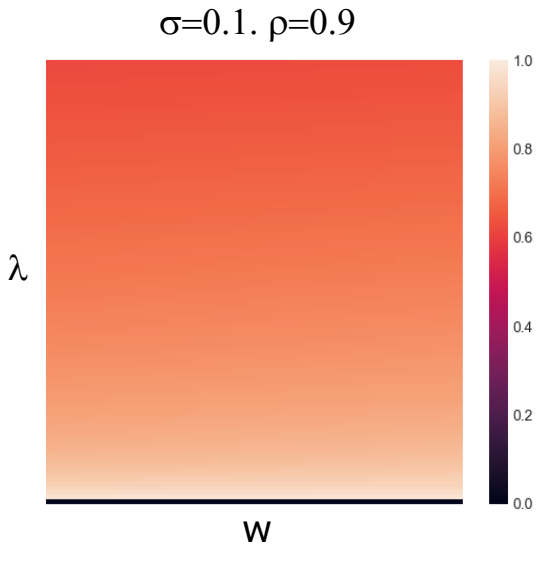


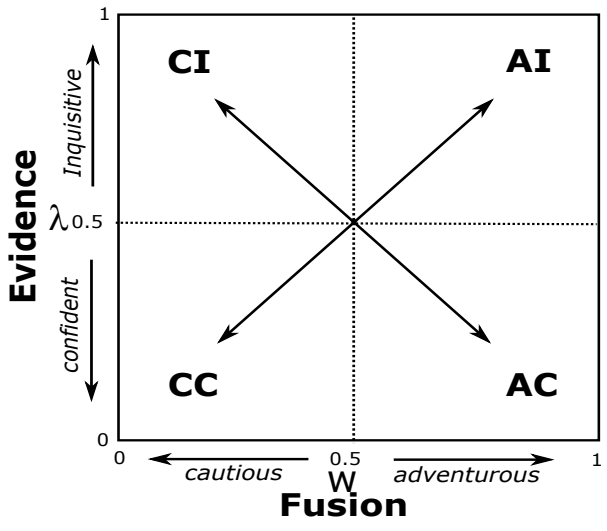
w AC & 1-w CC

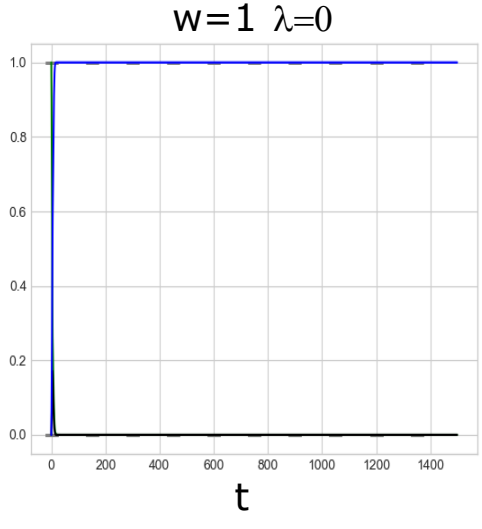


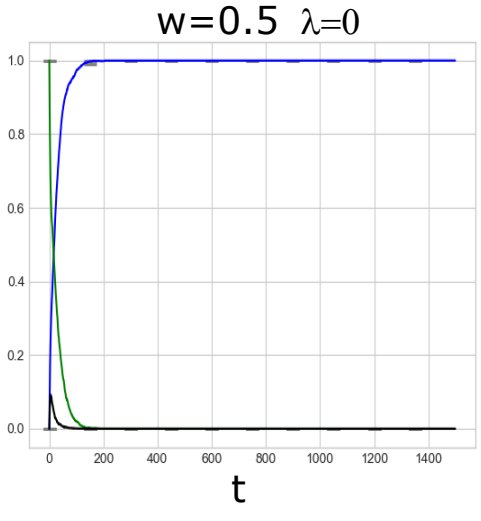
Mixtures & Dynamic Environments

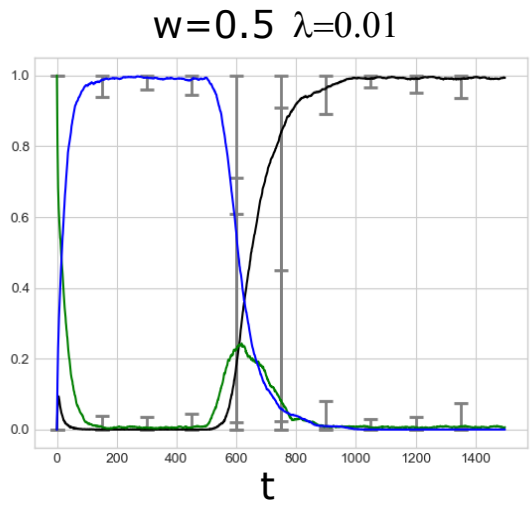












Summary

- We have presented social learning as a combination of two processes; fusion and updating.
- We have considered four overall behavioural types generated by independently combining conservative and open-minded approaches to both processes.
- Different behaviour types have been shown to have different convergence and consensus properties.
- Certain heterogeneous mixtures perform best in a range of different learning scenarios.
- A 50/50 mix of adventurous and cautious fusion combined with a mix of 1% inquisitive and 99% confident evidential updating, is highly robust especially in dynamic environments.